CAN PARTIAL HORIZONTAL OWNERSHIP LESSEN COMPETITION MORE THAN A MONOPOLY?

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Abstract

In this paper we investigate the anti-competitive effects of partial horizontal ownership in a setting where: (i) two cost-asymmetric firms compete à la Cournot; (ii) managers deal with eventual conflicting interests of the different shareholders by maximizing a weighted sum of firms’ operating profits; and (iii) weights result from the corporate control structure of the firm they run. Within this theoretical structure, we find that if the manager of the more efficient firm weights the operating profit of the (inefficient) rival more than its own profit, then partial ownership can lessen competition more than a monopoly.

JEL Classification: L11, L12, L13, L41, L50

Keywords: Partial Horizontal Ownership, Common-Ownership, Cross-Ownership, Full Joint Ownership, Duopoly, Cost Asymmetry

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1 Introduction

Horizontal shareholding exists when shareholders own partial ownership stakes in several horizontal competitors in an industry. This may induce a conflict of objectives among the owners of a firm, since a shareholder who - for example - also holds a stake in a rival firm typically wants the firm to pursue a less aggressive strategy than the strategy desired by a shareholder which does not hold a stake in a rival firm. In order to model this feature, the literature typically considers the strategy $x_j$ of firm $j$ in such cases to be decided and executed by a manager, who weights the (eventual) conflicting objectives of the different shareholders according to the corporate control structure of the firm (which determines the influence of each of those owners over decision-making), as follows:

$$\max_{x_j} \pi_j + \sum_{g \in \mathcal{G}, g \neq j} w_{jg} \pi_g,$$

where $\mathcal{G}$ denotes the set of existing firms in the industry, $\pi_j$ denotes the operating profit of firm $j$, and $w_{jg} \geq 0$ denotes the weight that the manager of firm $j$ assigns to the operating profit of each rival firm $g$.

The dominant formulation for the weight that the manager of a firm assigns to the operating profits of rivals is due to O’Brien and Salop (2000), who - rooted on Rotemberg (1984) and Bresnahan and Salop (1986) - assumes, the manager should decide the strategy of the firm so as to maximize a corporate control weighted sum of the firm’s shareholders returns. This yields that $w_{jg} = \frac{\sum_{k \in \Theta_j} \gamma_{kj} \phi_{kj}}{\sum_{k \in \Theta_j} \gamma_{kj}}$, where $\Theta_j$ denotes the set of shareholders that hold financial rights in firm $j$, $\phi_{kj}$ denotes the financial rights of shareholder $k$ in firm $j$, and $\gamma_{kj}$ denotes the control rights of shareholder $k$ in firm $j$. This formulation has been critiqued for producing counter-intuitive weights when non-horizontal shareholders are highly dispersed (see, for example, Gramlich and Grundl, 2017). Recently, Crawford et al. (2018) and Brito et al. (2018a) proposed alternative formulations to avoid this issue. Under these alternative formulations, the manager should decide the strategy of the firm so as to maximize a corporate control weighted sum of the shareholders relative returns. They yield that $w_{jg} = \sum_{k \in \Theta_j} \frac{\gamma_{kj} \phi_{kj}}{\phi_{kh}}$ and $w_{jg} = \sum_{k \in \Theta_j} \frac{\gamma_{kj} \phi_{kj}}{\phi_{kj}}$, respectively.\footnote{While the financial rights $\phi_{kj}$ have very clear empirical counterparts, the control rights $\gamma_{kj}$ do not. Azar (2016, 2017) and Brito et al. (2018a, 2018b) address this question by showing that O’Brien and Salop (2000) and Brito et al. (2018a)’s formulations can be microfounded through a probabilistic voting model in which shareholders vote to elect one of two candidates to the manager position, an incumbent and a challenger. In particular, they show that if the two candidates choose the strategy to the firm by maximizing the expected vote share within the firm, the control rights of shareholders can be endogenously measured by their holdings of voting stock. If, however, the two candidates maximize the probability of being elected, the control rights of shareholders can be endogenously measured by the Banzhaf (1965) power index that results from their holdings of voting stock.}
formulations can be extended to jointly capture horizontal shareholding by shareholders that can be external (common-ownership) and internal/rival firms (cross-ownership) to the industry. In those cases, the weights are computed using the *ultimate* financial and control rights, respectively, of *external* shareholders (see Brito *et al.*, 2018b).

Independently of the exact formulation, we have that, under partial ownership, managers would internalize the impact of their firm’s strategy on the operating profits of rivals when their firm’s controlling shareholders have financial rights in the rivals, which is confirmed by recent empirical work. For instance, Azar, Schmalz and Tecu (2018) examine the U.S. airline industry and find that the interlinks in the ownership of the airlines matter for how the airlines compete. Azar, Raina and Schmalz (2016) find the same relation in the U.S. banking industry. As a consequence, competition agencies have taken an increased interest in assessing the anti-competitive effects of partial horizontal ownership acquisitions. To do so, they may resort to Brito *et al.* (2018b)’s generalized Herfindahl-Hirschman index, which generalizes O’Brien and Salop (2000)’s modified index to measure the concentration of an industry characterized by partial common- and/or cross-ownership:

$$GHHI = \sum_{j \in \mathcal{S}} \sum_{g \in \mathcal{S}} \omega_{jg} s_j s_g,$$

where \(s_j\) and \(s_g\) denote the output market share of firms \(j\) and \(g\), respectively.

Since partial ownership do not completely and permanently eliminate competition among firms, we would expect, at first glance, prices in an industry characterized by partial ownership to be lower than in an industry characterized by a monopoly. However, the weights that managers assign to the operating profits of rivals (independently of the exact formulation) are not necessarily bounded from above, as illustrated by the following example, borrowed from Gramlich and Grundl (2017).\(^2\) Consider an industry with two firms: \(j\) and \(g\). Suppose that one (external) shareholder holds 5% financial and control rights in firm \(j\) and 45% in firm \(g\); and each of each firm’s remaining 1000 (external) shareholders holds equal financial and control rights in solely one firm. In this setting, the dominant formulation yields that \(\omega_{jg} \approx 6.61\) and \(\omega_{gj} \approx 0.11\). As a consequence, competition agencies may obtain concentration measures above 10,000. *Does this mean that prices in an industry characterized by partial ownership can be higher than in an industry characterized by a monopoly?*

To the best of our knowledge there is no article in the literature that examines this question. Nye (1992) and Foros, Kind and Shaffer (2011) examine the related sub-question

\(^2\)The weights that managers assign to the operating profits of rivals under the alternative formulation proposed by Brito *et al.* (2018a) are bounded above by one if control rights are measured by voting rights, but not if control rights are measured by the Banzhaf (1965) power index that results from the corresponding voting rights.
of whether prices in an oligopoly after a partial ownership acquisition can be higher than after a merger and show the answer can, in fact, be positive.\(^3\) In order to examine if prices in an industry characterized by partial ownership can be higher than even in an industry characterized by a monopoly, we consider a Cournot homogenous-product duopoly model in which demand is assumed to be linear and each firm is assumed to have constant asymmetric marginal cost and no capacity constraint. We consider also that the ownership structure is such that the manager of each firm weights the operating profit of the rival. We show that if the manager of the more efficient firm weights the operating profit of the (inefficient) rival more than its own profit, then partial ownership can lessen competition more than a monopoly, regardless of the weight the manager of the inefficient firm assigns to the operating profit of the rival.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model under which the Cournot-Nash industry equilibrium is derived, Section 3 discusses policy implications, and Section 4 concludes.

2 Theoretical Model

2.1 Setup

Consider a duopoly between firms \(j\) and \(g\) in a Cournot homogenous-product industry with no capacity constraint. Under this setting, we have that \(x_j = q_j\), with price \(p\) being determined by the downward sloping inverse market demand function, \(p(Q)\), where \(q_j\) denotes the quantity of firm \(j\) and \(Q = q_j + q_g\) denotes the industry total quantity. The market demand function is assumed to be linear: \(p(Q) = a - bQ\); and each firm is assumed to have constant marginal cost: \(c_j\) and \(c_g\). In order to examine the impact of cost asymmetries, let

\(^3\)Nye (1992) examines the 1989's Renault/Volvo joint venture in the heavy truck industry, which involved reciprocal partial ownership acquisitions by the two firms. As a consequence of this joint venture, the manager of Volvo assigned a weight of 0.55 to Renault's (heavy truck) operating profit while the manager of Renault assigned a weight of 0.505 to Volvo's (heavy truck) operating profit. Using data on market shares, Nye (1992) calibrates a Cournot homogenous-product oligopoly model in which demand is assumed to be linear and each firm is assumed to have constant asymmetric marginal cost and no capacity constraint. He then calculates that the joint venture would reduce industry total quantity by 1.91\% while a merger would reduce it solely by 1.71\%. However, he does not derive the conditions that sustain this result and (implicitly) assumes that an (unique) interior equilibrium exists. Foros, Kind and Shaffer (2011) examine a similar question under a Bertrand differentiated-product triopoly Salop model in which each firm is assumed to have identical costs of production and no capacity constraint. They compare the profitability of a merger between two of the firms and the profitability of a partial ownership acquisition (involving solely the same two firms) in which the acquiring firm, although acquiring less than 100\% of the financial rights, obtains full control. They show that a necessary condition for a partial ownership acquisition to be more profitable than a merger is that the equilibrium price of the outside firm increases when the acquisition stake decreases, which yields that consumers in aggregate end up worse off than they would have been if the firms had merged.
2.2 Best-Response Functions

Assuming that the ownership structure is such that the manager of firm $j$ places weight $w_{jg}$ on firm $g$’s operating profit and that the manager of firm $g$ places weight $w_{gj}$ on firm $j$’s operating profit, the manager of firm $j$ solves:

$$
\max_{q_j} \pi_j + w_{jg} \pi_g = (p - c_j) q_j + w_{jg} (p - \lambda c_j) q_g,
$$

(3)

whereas the manager of firm $g$ solves:

$$
\max_{q_g} \pi_g + w_{gj} \pi_j = (p - \lambda c_j) q_g + w_{gj} (p - c_j) q_j.
$$

(4)

The first-order conditions for $q_j$ and $q_g$ imply the following best-response functions for the two firms:

$$
BR_j : q_j = \begin{cases} \frac{(1-k_j) a}{2b} - \frac{(1+w_{jg})}{2} q_g & \text{if } q_g < \frac{(1-k_j) a}{(1+w_{jg}) b} \\ 0 & \text{if } q_g \geq \frac{(1-k_j) a}{(1+w_{jg}) b} \end{cases}
$$

(5)

$$
BR_g : q_g = \begin{cases} \frac{(1-k_j) a}{2b} - \frac{(1+w_{gj})}{2} q_j & \text{if } q_j < \frac{(1-k_j) a}{(1+w_{gj}) b} \\ 0 & \text{if } q_j \geq \frac{(1-k_j) a}{(1+w_{gj}) b} \end{cases}
$$

where $k_j = \frac{c_j}{a} < 1$. Note that since $a > \lambda c_j$, we have that $\lambda k_j < 1$. The effects involved in the best-response functions are as follows. By producing one more unit, firm $j$ (for example) receives the corresponding profit margin. However, by doing so, it decreases market price, which lowers the profit margin of all firm $j$’s inframarginal units and of firm $g$’s quantity. To the extent that the manager of firm $j$ cares for the operating profit of firm $g$ this effect is internalized (partially when $w_{jg} < 1$, fully when $w_{jg} = 1$ or "more than fully" when $w_{jg} > 1"). Thus, the higher the weight placed on the rival’s operating profit, the lower the equilibrium quantity.

2.3 Industry Total Quantity Equilibria

Depending on where the best responses cross the horizontal and vertical axis, we can identify seven Cournot-Nash equilibria types. Let $\bar{w}_{jg} = \frac{1+k_j(\lambda-2)}{1-\lambda k_j} > 1$ and $\bar{w}_{gj} = \frac{1-k_j(2\lambda-1)}{1-k_j} < 1$. This implies that the best response of firm $j$ crosses the vertical axis above the best response of firm $g$ whenever $\frac{(1-k_j) a}{(1+w_{jg}) b} > \frac{(1-\lambda k_j) a}{2b}$ or $w_{jg} < \bar{w}_{jg}$ while it crosses the horizontal axis to
the right of the best response of firm \( g \) whenever \( \frac{(1-k_j)a}{(1+a_{kj})b} > \frac{(1-k_j)a}{2b} \) or \( w_{gj} < \bar{w}_{gj} \). Figure 1 depicts the different equilibria types, which are characterized in Lemma 1.

**Lemma 1**

(a) If \( w_{jg} < \bar{w}_{jg} \) and \( w_{gj} < \bar{w}_{gj} \), there is an unique Cournot-Nash interior equilibrium, given by:

\[
q_{j}^{po} = \frac{((1 - w_{jg}) - (2 - \lambda (1 + w_{jg})) k_j) a}{(3 - w_{gj} - w_{jg} - w_{gj} w_{jg}) b}
\]

\[
q_{g}^{po} = \frac{((1 - w_{gj}) - (2\lambda - 1 - w_{gj}) k_j) a}{(3 - w_{gj} - w_{jg} - w_{gj} w_{jg}) b}
\]

(b) If \( w_{jg} < \bar{w}_{jg} \) and \( w_{gj} \geq \bar{w}_{gj} \), there is an unique Cournot-Nash corner equilibrium, given by \( q_{j}^{po} = \frac{(1-k_j)a}{2b} \) and \( q_{g}^{po} = 0 \).

(c) If \( w_{jg} \geq \bar{w}_{jg} \) and \( w_{gj} < \bar{w}_{gj} \), there is an unique Cournot-Nash corner equilibrium, given by \( q_{j}^{po} = 0 \) and \( q_{g}^{po} = \frac{(1-k_j)a}{2b} \).

(d) If \( w_{jg} > \bar{w}_{jg} \) and \( w_{gj} > \bar{w}_{gj} \), there are three Cournot-Nash equilibria, given by all the pairs of quantities described in (a) to (c).

(e) If \( w_{jg} = \bar{w}_{jg} \) and \( w_{gj} = \bar{w}_{gj} \), there are multiple Cournot-Nash interior and corner equilibria, given by any pair \( q_{j}^{po}, q_{g}^{po} \) such that \( q_{j}^{po} = \frac{(1-k_j)a}{2b} - \frac{1-k_j}{1-k_j} q_{j}^{po} \).

(f) If \( (w_{jg} = \bar{w}_{jg} \) and \( w_{gj} > \bar{w}_{gj}) \) or \( (w_{jg} > \bar{w}_{jg} \) and \( w_{gj} = \bar{w}_{gj}) \), there are two Cournot-Nash corner equilibria, given by all the pairs of quantities described in (b) and (c).

**Proof.** The result follows directly from intersecting the best-response functions for the different parameter values.

### 2.3.1 Monopoly

Under a monopoly, we have \( w_{jg} = w_{gj} = 1 \), which implies that the manager of each firm fully internalizes the impact of their firm’s strategy on the rival firm operating profits. Since \( \bar{w}_{jg} > 1 \) and \( \bar{w}_{gj} < 1 \), this corresponds to part (b) of Lemma 1: \( w_{jg} = 1 < \bar{w}_{jg} \) and \( w_{gj} = 1 > \bar{w}_{gj} \). As a consequence, we have that the Cournot-Nash equilibrium is unique and given by \( q_{j}^{m} = \frac{(1-k_j)a}{2b} \) and \( q_{g}^{m} = 0 \), yielding that the industry total quantity is given by \( Q^{m} = q_{j}^{m} + q_{g}^{m} = \frac{(1-k_j)a}{2b} \). In other words, in a monopoly involving the two firms, managers assign production to the more efficient firm.

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\(^{4}\)The interior equilibrium described in part (d) is not, however, stable (see Tirole, 1988).
Figure 1
Cournot-Nash Equilibria Types
2.3.2 Partial Ownership vs Monopoly

Having established the Cournot-Nash equilibria for the two firms, we can now examine how the industry total quantity under partial ownership and monopoly, compares. We begin by addressing the corner equilibria under partial ownership. In the equilibria in which only firm \( j \), the most efficient firm, produces, the industry total quantity under partial ownership is given by

\[
Q_{po} = q_{po}^j + q_{po}^g = \left(1-k_j\right)a = Q^m,
\]

which is the same as under monopoly. This equilibria is sustainable for \( w_{gj} \geq \bar{w}_{gj} \), regardless of \( w_{jg} \), as depicted in Figure 2, Panel (a).

We now address the corner equilibria in which only firm \( g \), the inefficient firm, produces, the industry total quantity under partial ownership is given by

\[
Q_{po} = q_{po}^j + q_{po}^g = \frac{\left(1-k_g\right)a}{2b} < Q^m,
\]

which is strictly lower than the industry total quantity under (an efficient) monopoly. This equilibria is sustainable for \( w_{jg} \geq \bar{w}_{jg} \), regardless of \( w_{gj} \), as depicted in Figure 2, Panel (b).

Finally, we address the interior equilibria, which is characterized in Proposition 1 and
depicted in Figure 2, Panel (c).

**Proposition 1** In an interior equilibrium under partial ownership, the industry total quantity is:

(a) strictly higher than under monopoly if \( w_{jg} < 1 \) and \( w_{gj} < w_{gj} \).

(b) the same as under monopoly if \( w_{jg} = 1 \) and \( w_{gj} < w_{gj} \).

(c) strictly lower than under monopoly if \((1 < w_{jg} < \bar{w}_{jg} \text{ and } w_{gj} < \bar{w}_{gj}) \text{ or } (w_{jg} = \bar{w}_{jg} \text{ and } w_{gj} = \bar{w}_{gj}) \text{ or } (w_{jg} > \bar{w}_{jg} \text{ and } w_{gj} > \bar{w}_{gj})\).

**Proof.** See Appendix. ■

The intuition for the different interior equilibria is as follows. In the case of monopoly, the industry total quantity corresponds to the monopoly quantity by the most efficient firm, firm \( j \), which is obviously the best response of firm \( j \) to zero output by firm \( g \). In an interior equilibrium with partial ownership, firm \( g \)'s quantity will be strictly positive. It is only possible that this leads to a strictly higher (or the same) industry total quantity than (as) the one under monopoly if the best response of firm \( j \) (to the increased quantity by firm \( g \)) is to lower its own output by less (or the same) than (as) firm \( g \) increased it. This means that the slope of firm \( j \)'s best response function must be higher (or equal) than -1 and for this to happen, the manager of firm \( j \) must weight firm \( g \)'s operating profit less (or the same) than (as) its own, i.e. \( w_{jg} \leq 1 \). This implies that when the manager of firm \( j \) weights firm \( g \)'s operating profit more than its own, i.e. \( w_{jg} > 1 \), in an interior equilibrium with partial ownership, the industry total quantity will be strictly lower than under monopoly.

**Cost Asymmetry** We now discuss the role played by cost asymmetry in the comparison discussed above. To do so, note that \( \frac{\partial \bar{w}_{jg}}{\partial \lambda} = \frac{2 k_j (1-k_j)}{(1-k_j)^2} > 0 \) and \( \frac{\partial \bar{w}_{gj}}{\partial \lambda} = \frac{-2 k_j}{1-k_j} < 0 \). This has several implications for the equilibria under partial ownership. First, an increase in the inefficiency level of firm \( g \) reduces the set of values for \( w_{jg} \) that sustain a corner equilibrium in which only firm \( g \), the inefficient firm, produces, yielding an industry total quantity under partial ownership that is strictly lower than under (an efficient) monopoly. Second, an increase in the inefficiency level of firm \( g \) increases the set of values for \( w_{gj} \) that sustain a corner equilibrium in which only firm \( j \), the most efficient firm, produces, yielding an industry total quantity under partial ownership that is the same as under monopoly. Third, an increase in the inefficiency level of firm \( g \) reduces the set of values for \( w_{gj} \) that sustain an interior equilibrium in which the industry total quantity under partial ownership is strictly
higher than under monopoly. Fourth, an increase in the inefficiency level of firm \( g \) reduces the set of values for \( w_{gj} \) that sustain an interior equilibrium in which the industry total quantity under partial ownership is the same as under monopoly. Finally, an increase in the inefficiency level of firm \( g \) has an ambiguous effect on the set of values for \( w_{jg} \) and \( w_{gj} \) that sustain an interior equilibrium in which the industry total quantity under partial ownership is strictly lower than under monopoly.

3 Policy Implications

Corollary 1 summarizes the equilibria discussed in the previous section with respect to how the industry total quantity under partial ownership and monopoly, compares.

**Corollary 1** The equilibrium industry total quantity under partial ownership is:

(a) strictly higher than or the same as under monopoly if \( w_{jg} \leq 1 \).

(b) strictly lower than or the same as under monopoly if \( w_{jg} > 1 \).

**Proof.** See Appendix. □

This implies that if the manager of the more efficient firm does not weight the operating profit of the (inefficient) rival more than its own profit, then partial ownership can not lessen competition more than a monopoly while if the manager of the more efficient firm weights the operating profit of the (inefficient) rival more than its own profit, then partial ownership can lessen competition more than a monopoly. This occurs regardless of the weight the manager of the inefficient firm assigns to the operating profit of the rival. As a consequence, when assessing the anti-competitive effects of acquisitions that give raise to common- or cross-ownership, competition agencies should examine with increased interest the objective function of the manager of the most efficient firm in the industry.

4 Conclusions

We propose to examine whether prices in an industry characterized by partial ownership can be higher than in an industry characterized by a monopoly. To do so, we consider a Cournot duopoly model in which demand is assumed to be linear and each firm is assumed to have constant asymmetric marginal cost and no capacity constraint. We consider also that the ownership structure is such that the manager of each firm weights the operating profit
of the rival. We show that if the manager of the more efficient firm weights the operating profit of the rival more than its own profit, then partial ownership can lessen competition more than a monopoly. This article leaves many other settings yet to be explored. In particular, extensions that examine the value of the generalized Herfindahl-Hirschman index in equilibrium and/or consider settings with more than two firms constitute very interesting potential areas for future research. Hopefully, however, our contribution can be seen as a stepping stone in the direction of a more complete analysis.

References


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Appendix

In this appendix, we present the proofs of Proposition 1 and of Corollary 1.

**Proof of Proposition 1.** Let \( f(w_{gj}) = \frac{3-w_{gj}}{1+w_{gj}} \) with \( \frac{\partial f(w_{gj})}{\partial w_{gj}} < 0 \), \( f(1) = 1 \) and \( f(\bar{w}_{gj}) = \bar{w}_{gj} \).

(a) If \( w_{gj} < 1 < \bar{w}_{gj} \) and \( w_{gj} < \bar{w}_{gj} \), the industry total quantity under partial ownership is strictly higher than under monopoly if and only if \( Q^{po} > Q^m \):

\[
\frac{(2 - w_{gj} - w_{jg}) - k_j (1 + \lambda w_{gj} - w_{gj})}{(3 - w_{gj} - w_{jg} - w_{gj} w_{jg})} > \frac{1-k_j}{2},
\]

which is equivalent to:

\[
\frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} > 0.
\]

Note that we have \( (w_{gj} - \bar{w}_{gj}) < 0 \) and \( f(w_{gj}) > w_{jg} \) because \( w_{gj} < \bar{w}_{gj} \) implies \( f(w_{gj}) > f(\bar{w}_{gj}) = \bar{w}_{jg} > w_{jg} \). Then, \( \frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} > 0 \) if \( (w_{gj} - 1) < 0 \), which is equivalent to \( w_{jg} < 1 \).

(b) If \( w_{jg} = 1 < \bar{w}_{jg} \) and \( w_{gj} < \bar{w}_{gj} \), the industry total quantity under partial ownership is the same as under monopoly if and only if \( Q^{po} = Q^m \), which - as discussed above - is equivalent to:

\[
\frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} = 0.
\]

Note that we have \( (w_{gj} - \bar{w}_{gj}) < 0 \) and \( f(w_{gj}) > w_{jg} \) because \( w_{gj} < \bar{w}_{gj} \) implies \( f(w_{gj}) > f(\bar{w}_{gj}) = \bar{w}_{jg} > w_{jg} \). Then, \( \frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} = 0 \) if \( (w_{jg} - 1) = 0 \), which is equivalent to \( w_{jg} = 1 \).

(c) We need to evaluate three cases:

i. If \( 1 < w_{jg} < \bar{w}_{jg} \) and \( w_{gj} < \bar{w}_{gj} \), the industry total quantity under partial ownership is strictly lower than under monopoly if and only if \( Q^{po} < Q^m \), which - as discussed above - is equivalent to:

\[
\frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} < 0.
\]

Note that we have \( (w_{gj} - \bar{w}_{gj}) < 0 \) and \( f(w_{gj}) > w_{jg} \) because \( w_{gj} < \bar{w}_{gj} \) implies \( f(w_{gj}) > f(\bar{w}_{gj}) = \bar{w}_{jg} > w_{jg} \). Then, \( \frac{(w_{gj} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{jg}} > 0 \) if \( (w_{jg} - 1) > 0 \), which is equivalent to \( w_{jg} > 1 \).
ii. If \( w_{ij} = \bar{w}_{ij} \) and \( w_{gj} = \bar{w}_{gj} \) and an interior equilibrium exists, it would be characterized by a pair \( q_{po}^j > 0, q_{po}^g > 0 \) such that \( q_{po}^j = \frac{(1-\lambda k_j)u}{2b} - \frac{1-\lambda k_j}{1-k_j} q_{po}^g \). Note that \( \frac{(1-\lambda k_j)u}{2b} > 0 \) and \( 0 < \frac{1-\lambda k_j}{1-k_j} < 1 \). This implies that the industry total quantity under partial ownership is strictly lower than under monopoly: \( \frac{(1-\lambda k_j)u}{2b} < q_{po}^j < \frac{(1-\lambda k_j)u}{2b} = Q^m \).

iii. If \( w_{ij} > \bar{w}_{ij} \) and \( w_{gj} > \bar{w}_{gj} \) and an interior equilibrium exists, the industry total quantity under partial ownership is strictly lower than under monopoly if and only if \( Q_{po} < Q^m \), which - as discussed above - is equivalent to:

\[
\frac{(w_{ij} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{ij}} < 0.
\]

Note that we have \( (w_{ij} - \bar{w}_{gj}) > 0 \) and \( f(w_{gj}) < w_{ij} \) because \( w_{ij} > \bar{w}_{gj} \) implies \( f(w_{gj}) < f(\bar{w}_{gj}) = \bar{w}_{ij} < w_{ij} \). Then, \( \frac{(w_{ij} - 1)(w_{gj} - \bar{w}_{gj})}{f(w_{gj}) - w_{ij}} < 0 \), which is equivalent to \( (w_{ij} - 1) > 0 \) which, in turn, is always true, as \( w_{ij} > \bar{w}_{ij} > 1 \).

**Proof of Corollary 1.**

(a) We need to evaluate three cases:

i. If \( w_{ij} < 1 < \bar{w}_{ij} \) and \( w_{gj} < \bar{w}_{gj} \), there is, as established by part (a) of Lemma 1, an interior equilibrium in which the industry total quantity under partial ownership is, as established by part (a) of Proposition 1, strictly higher than under monopoly.

ii. If \( w_{ij} = 1 < \bar{w}_{ij} \) and \( w_{gj} < \bar{w}_{gj} \), there is, as established by part (a) of Lemma 1, an interior equilibrium in which the industry total quantity under partial ownership is, as established by part (b) of Proposition 1, the same as under monopoly.

iii. If \( w_{ij} \leq 1 < \bar{w}_{ij} \) and \( w_{gj} \geq \bar{w}_{gj} \), there is, as established by part (b) of Lemma 1, a corner equilibrium in quantities in which only firm \( j \), the most efficient firm, produces. As such, the industry industry total quantity under partial ownership is the same as under monopoly.

(b) We need to evaluate six cases:

i. If \( 1 < w_{ij} < \bar{w}_{ij} \) and \( w_{gj} < \bar{w}_{gj} \), there is, as established by part (a) of Lemma 1, an interior equilibrium in which the industry total quantity under partial ownership is, as established by part (c) of Proposition 1, strictly lower than under monopoly.

ii. If \( 1 < w_{ij} < \bar{w}_{ij} \) and \( w_{gj} \geq \bar{w}_{gj} \), there is, as established by part (b) of Lemma 1, a corner equilibrium in quantities in which only firm \( j \), the most efficient firm, produces. As such, the industry industry total quantity under partial ownership is the same as under monopoly.
iii. If $w_{jg} \geq \bar{w}_{jg} > 1$ and $w_{gj} < \bar{w}_{gj}$, there is, as established by part (c) of Lemma 1, a corner equilibrium in quantities in which only firm $g$, the inefficient firm, produces. As such, the industry industry total quantity under partial ownership is strictly lower than under (an efficient) monopoly.

iv. If $w_{jg} > \bar{w}_{jg} > 1$ and $w_{gj} > \bar{w}_{gj}$, there is, as established by part (d) of Lemma 1, three equilibria in quantities. A corner equilibrium in quantities in which only firm $j$, the most efficient firm, produces and, as such, the industry industry total quantity under partial ownership is the same as under monopoly. A corner equilibrium in quantities in which only firm $g$, the inefficient firm, produces and, as such, the industry industry total quantity under partial ownership is strictly lower than under (an efficient) monopoly. An interior equilibrium in which the industry total quantity under partial ownership is, as established by part (c) of Proposition 1, strictly lower than under monopoly.

v. If $w_{jg} = \bar{w}_{jg} > 1$ and $w_{gj} = \bar{w}_{gj}$, there is, as established by part (e) of Lemma 1, multiple equilibria in quantities characterized by a pair $d_{j}^{po}, q_{g}^{po}$ such that $d_{g}^{po} = \frac{(1-k_{j})a}{2b} - \frac{1-k_{j}}{1-k_{j}} d_{j}^{po}$. This yields two corner equilibria and multiplicity of interior equilibria. The two corner equilibria are as follows. A corner equilibrium in quantities in which only firm $j$, the most efficient firm, produces and, as such, the industry industry total quantity under partial ownership is the same as under monopoly. A corner equilibrium in quantities in which only firm $g$, the inefficient firm, produces and, as such, the industry industry total quantity under partial ownership is strictly lower than under (an efficient) monopoly. In all interior equilibria, the industry total quantity under partial ownership is, as established by part (c) of Proposition 1, strictly lower than under monopoly.

vi. If $(w_{jg} = \bar{w}_{jg} > 1$ and $w_{gj} > \bar{w}_{gj})$ or $(w_{jg} > \bar{w}_{jg} > 1$ and $w_{gj} = \bar{w}_{gj})$, there is, as established by part (f) of Lemma 1, two corner equilibria in quantities. A corner equilibrium in quantities in which only firm $j$, the most efficient firm, produces and, as such, the industry industry total quantity under partial ownership is the same as under monopoly. A corner equilibrium in quantities in which only firm $g$, the inefficient firm, produces and, as such, the industry industry total quantity under partial ownership is strictly lower than under (an efficient) monopoly.