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MODELING HORIZONTAL SHAREHOLDING WITH OWNERSHIP DISPERSION

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Modeling Horizontal Shareholding with Ownership Dispersion*

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Abstract

The dominant formulation for modeling the objective function of managers of competing firms with horizontal shareholding has been critiqued for producing the result that, if non-horizontal shareholders are highly dispersed, managers would mimic the interests of horizontal shareholders even if they own a share of the firm that does not induce full control. We show that this issue can be avoided (while maintaining the remaining features of the dominant approach) with an alternative formulation that is derived from a probabilistic voting model that assumes shareholders with higher financial stakes will take greater interest in the managerial actions, which yields the result that managers maximize a control-weighted sum of the shareholders' relative returns.

JEL Classification: L13, L41

Keywords: Horizontal Shareholding, Ownership Dispersion, Manager Objective Function, Proportional Control, Banzhaf Control

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1 Introduction

Horizontal shareholding is common ownership in competing firms. Such horizontal shareholding can induce a conflict in the firm-specific objectives of shareholders, wherein horizontal shareholders in any given firm want that firm to pursue a less competitive strategy than the strategy desired by non-horizontal shareholders.[1] Hence, firm managers must weigh the conflicting objectives of different shareholders according to their relative influence over firm decision-making.

Schmalz (2018) discusses the desirable properties for the weighting scheme used by managers: (*i*) absent horizontal shareholding, managers would maximize their firm's own profit; (*ii*) with horizontal shareholding, managers would internalize the impact of their firm's strategy on rival firm profits when their firm's controlling shareholders have financial rights in the rival; (*iii*) the weight that managers assign to rival firms would be continuous on the financial and control rights of the firm's shareholders; (*iv*) managers would maximize industry profit when all controlling shareholders are fully diversified across rivals; and (*v*) the weight that managers assign to rival firm profits would reflect relatively more the interests of relatively large shareholders. Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford *et al.* (2018) discuss an additional property: (*vi*) the weight that managers assign to rival firm profits would not mimic the interests of horizontal shareholders when they own a share of the firm that, even if non-horizontal shareholders are highly dispersed, does not induce full control.

The dominant formulation of the objective function of managers is due to O'Brien and Salop (2000, henceforth O&S), who incorporating features from both Rotemberg (1984) and Bresnahan and Salop (1986), assume *the manager would decide the strategy of the firm to maximize a control-weighted sum of the firm's shareholders returns.* Because those returns are a function of the profits of the firms in which shareholders hold financial rights, this implies that the manager of any firm j would maximize a weighted sum of the profits of (potentially) all the firms in the industry:

$$\max_{x_j} \sum\nolimits_{k\in\Theta} \gamma_{kj} R_k = \sum\nolimits_{k\in\Theta} \gamma_{kj} \left(\sum\nolimits_{g\in\Im} \phi_{kg}\pi_g\right) \propto \max_{x_j} \pi_j + \sum\nolimits_{g\in\Im, g\neq j} \frac{\sum_{k\in\Theta} \gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta} \gamma_{kj}\phi_{kj}} \pi_g, \quad (1)$$

where Θ and $\Im$ denote the set of existing shareholders and firms, respectively, x_j denotes the strategy of firm j, γ_{kj} denotes the control rights of shareholder k in firm j, $R_k = \sum_{g\in\Im} \phi_{kg}\pi_g$

[1] Although non-horizontal shareholders may favor a different firm-specific strategy, that does not mean they are harmed by horizontal shareholding because horizontal shareholding also reduces the competitiveness of rival firms, and non-horizontal shareholders benefit from a mutual reduction of competition at both the firm and its rivals.

denotes the returns of shareholder k, ϕ_{kj} denotes the financial rights of shareholder k in firm j, and π_j denotes the operating profit of firm j. Azar (2016, 2017) shows that this formulation can be microfounded through a probabilistic voting model in which shareholders elect one of two potential managers, which yields that the control rights of each shareholder will (a) if managers maximize their vote share, be proportional to their voting rights (proportional control) and (b) if managers maximize their odds of election, equal the odds that their vote will be pivotal in the election (i.e, by their Banzhaf (1965) power index).[2,3]

However, some critique the dominant formulation for failing property (vi).[4] See Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford *et al.* (2018). In this paper, we propose an alternative formulation. In the lines of Azar (2016, 2017) and Brito *et al.* (2018), we use a probabilistic voting model. But unlike prior literature, we assume that managers expect shareholders with higher financial stakes in their firm will (for example) incur more effort to become informed on the vote and thus could potentially have a larger preference for (or against) the challenger than other shareholders. Then, in equilibrium, the manager chooses the strategy of the firm to maximize a control-weighted sum of the firm's shareholders *relative* returns:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\tilde{R}_k = \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\left(\sum\nolimits_{g\in\Im} \frac{\phi_{kg}}{\phi_{kj}}\pi_g\right) = \pi_j + \sum\nolimits_{g\in\Im, g\neq j} \sum\nolimits_{k\in\Theta_j} \frac{\gamma_{kj}\phi_{kg}}{\phi_{kj}}\pi_g, \quad (2)$$

where Θ_j denotes the subset of shareholders who hold financial rights in firm j, $\tilde{R}_k$ denotes the *relative* returns of shareholder k, normalized by her financial rights in firm j. The intuition is as follows. The strategy proposals of the candidates impact the return of the firm's shareholders, which in turn impacts their probability of voting for the candidates. The assumption above implies that the latter impact is lower for shareholders with higher financial stakes since, having a larger preference for (or against) the challenger, they already have a larger probability of voting in one direction. As a consequence, candidates pay less attention to those shareholders than they would under the dominant formulation. They do so, by weighting not the absolute, but the relative returns of shareholders.[5]

Our proposed alternative formulation is similar in nature to the formulation in Crawford

[2] Azar (2017) also considers a probabilistic voting model in which shareholders vote either on whether to approve a manager-proposed change in the firm's strategic plan.

[3] Brito *et al.* (2018) generalize Azar (2016, 2017)'s framework to jointly capture common-ownership and cross-ownership by rival firms.

[4] In fact, as we show below, property (v) may - in certain cases - also fail under the dominant formulation.

[5] This means that if a shareholder owns a portfolio that is equal to another shareholder's portfolio multiplied by α, the manager will consider they both have the same relative returns. Their control rights will naturally be different, but their relative returns will be the same. This makes the smaller shareholder more relevant in the manager's objective function because in the dominant formulation, this shareholder would have smaller control rights and also smaller absolute returns.

et al. (2018) who, to address property (*vi*), normalize shareholder k's returns by $\sum_{h \in \Im} \phi_{kh}$, but we microfound our function through a probabilistic voting model. This alternative formulation can satisfy the six desirable properties discussed above.

2 An Illustrative Example

We now address an illustrative example, borrowed from Gramlich and Grundl (2017), to examine how the two formulations compare in terms of properties (*i*) to (*vi*). Imagine a duopoly in which one shareholder holds symmetric financial and voting rights in both firms and each of the remaining shareholders holds equal financial and voting rights in solely one firm. In this setting, properties (*i*) to (*iv*) clearly hold in both formulations. Online Appendix A examines properties (*v*) and (*vi*), which we now discuss.

If we combine O&S's formulation with an assumption of proportional control, property (*v*) holds. The weight that managers assign to rival firm profits increases when the relative size of the horizontal shareholder stakes increase (due to either higher levels of horizontal shareholding or higher number of non-horizontal shareholders). However, this combination also predicts that managers would engage in near-monopoly pricing when the non-horizontal shareholders are highly dispersed, even if the horizontal shareholder does not have full control, which fails property (*vi*). Likewise, if we combine O&S's formulation with an assumption of Banzhaf control, property (*v*) holds for increasing levels of horizontal shareholding with a constant number of shareholders, but may or may not hold for a constant level of horizontal shareholding with an increasing number of shareholders, since the number of subsets in which the horizontal shareholder is pivotal can decrease as the number of non-horizontal shareholders increase. However, again, property (*vi*) does not hold. Table 1, Panel A illustrates these features.

If we combine our proposed alternative formulation with an assumption of proportional control, property (*v*) holds for increasing levels of horizontal shareholding with a constant number of shareholders, but not for a constant level of horizontal shareholding with an increasing number of shareholders. The reason being that - in our example - the weight that the manager of each firm assigns to the profit of the rival firm is solely given by the control rights of the horizontal shareholder, since her financial rights in the two firms exactly cancel. However, this combination does not predict that managers would engage in near-monopoly pricing when the non-horizontal shareholders are highly dispersed, even if the horizontal shareholder does not have full control. It thus satisfies property (*vi*). Likewise, if we combine our formulation with an assumption of Banzhaf control, property (*v*) holds for increasing levels of horizontal shareholding with a constant number of shareholders, but may

or may not hold for a constant level of horizontal shareholding with an increasing number of shareholders (for the same reasons as discussed above). However, property (*vi*) holds. Table 1, Panel B illustrates these features.

3 The Theoretical Framework

The theoretical framework is based in Azar (2016, 2017) and Brito *et al.* (2018). There are K shareholders, indexed by $k \in \Theta$ and N firms, indexed by $j \in \Im$. The holdings of total stock of shareholder k in firm j, represented by $0 \leq \phi_{kj} \leq 1$ with $\sum_{k\in\Theta} \phi_{kj} = 1$, capture her *financial rights* to the firm's stream of profits. The holdings of voting stock of shareholder k in firm j, represented by $0 \leq \upsilon_{kj} \leq 1$ with $\sum_{k\in\Theta} \upsilon_{kj} = 1$, capture her *voting rights* in the firm that may not coincide with her control rights in the firm, which refer to the rights to influence the firm's decisions in a way discussed below.

We follow Azar (2016, 2017) in assuming a standard theory of probabilistic voting. We also assume, along Lindbeck and Weibull (1987), that the manager of each firm is the winner in an election between two candidates, an incumbent and a challenger, who compete for the shareholders' votes by proposing a strategy for the firm. Shareholders and candidates are assumed to play a two-stage game. First, candidates simultaneously choose their strategy proposals (e.g., quantity, price, etc.). Second, shareholders vote to elect their managers.

We assume the following regarding the voting behavior of shareholders:

Assumption 1 *Shareholders are conditionally sincere.*

Assumption 1 implies, following Alesina and Rosenthal (1995), that firm j's shareholders vote for the candidate whose strategy proposal maximizes their utilities, given the equilibrium strategy proposals of the candidates to the remaining firms, randomizing between them when indifferent.

We consider that the utility of shareholder k is a function of the winning strategies of all firms and involves two elements, assumed additively separable, as follows:

$$u_k = R_k + \sum\nolimits_{g\in\Im} d_g \xi_{kg} = \sum\nolimits_{g\in\Im} \phi_{kg}\pi_g + \sum\nolimits_{g\in\Im} d_g \xi_{kg} \qquad (3)$$

The first utility element follows from O&S and captures the utility associated to the *return of shareholder k's financial rights holdings.* The second utility element follows from Kramer (1983) and captures the utility associated to the *credibility (or lack of credibility) attached to the challenger' strategy proposal*, where d_g denotes a dummy variable that takes value 1 if the challenger is elected manager of firm g and ξ_{kg} denotes the utility that shareholder

Table 1

*Weight that each Manager Assigns to the Profit of the Rival Firm**

	Number of Non-Horizontal Shareholders					
	1	100	499	500	501	1000
Panel A: O&S's Formulation						
1% Horizontal Shareholding						
Proportional Control	0.000	0.010	0.048	0.049	0.049	0.093
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
Banzhaf Control	0.000	0.052	0.296	0.312	0.296	0.597
	(0.000)	(0.051)	(0.077)	(0.082)	(0.077)	(0.128)
5% Horizontal Shareholding						
Proportional Control	0.003	0.217	0.580	0.581	0.581	0.735
	(0.050)	(0.050)	(0.050)	(0.050)	(0.050)	(0.050)
Banzhaf Control	0.000	0.222	0.687	0.703	0.686	0.885
	(0.000)	(0.051)	(0.077)	(0.082)	(0.077)	(0.128)
10% Horizontal Shareholding						
Proportional Control	0.012	0.552	0.860	0.861	0.861	0.925
	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)
Banzhaf Control	0.000	0.650	0.986	0.985	0.986	1.000
	(0.000)	(0.143)	(0.564)	(0.534)	(0.560)	(0.950)
20% Horizontal Shareholding						
Proportional Control	0.059	0.862	0.969	0.969	0.969	0.984
	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)
Banzhaf Control	0.000	0.986	1.000	1.000	1.000	1.000
	(0.000)	(0.744)	(1.000)	(1.000)	(1.000)	(1.000)
Panel B: Proposed Alternative Formulation						
1% Horizontal Shareholding						
Proportional Control	0.010	0.010	0.010	0.010	0.010	0.010
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
Banzhaf Control	0.000	0.051	0.077	0.082	0.077	0.128
	(0.000)	(0.051)	(0.077)	(0.082)	(0.077)	(0.128)
5% Horizontal Shareholding						
Proportional Control	0.050	0.050	0.050	0.050	0.050	0.050
	(0.050)	(0.050)	(0.050)	(0.050)	(0.050)	(0.050)
Banzhaf Control	0.000	0.051	0.077	0.082	0.077	0.128
	(0.000)	(0.051)	(0.077)	(0.082)	(0.077)	(0.128)
10% Horizontal Shareholding						
Proportional Control	0.100	0.100	0.100	0.100	0.100	0.100
	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)
Banzhaf Control	0.000	0.143	0.564	0.534	0.560	0.950
	(0.000)	(0.143)	(0.564)	(0.534)	(0.560)	(0.950)
20% Horizontal Shareholding						
Proportional Control	0.200	0.200	0.200	0.200	0.200	0.200
	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)
Banzhaf Control	0.000	0.744	1.000	1.000	1.000	1.000
	(0.000)	(0.744)	(1.000)	(1.000)	(1.000)	(1.000)

* Please see Online Appendix A for the computational details. Control rights of the horizontal shareholder in parenthesis. The weights in Panel B exactly coincide with the control rights of the horizontal shareholder since - in our example - her financial rights in the two firms exactly cancel.

k obtains from such event. This implies that the shareholder's choice is deterministic and it is a discontinuous function of the difference in the utilities obtained from the strategy proposals of each candidate.

We follow Lindbeck and Weibull (1987) in assuming that the utility associated to the credibility of the challenger' strategy proposal, while known to voters, is unobserved by candidates, which treat it as a random utility shock independently distributed across firms and shareholders according to a symmetric probability distribution with mean zero and cumulative distribution $G_{kj}(\cdot)$.[6] Thus, from the candidates' perspective, voting by shareholders is probabilistic.

We make the following alternative assumptions regarding the candidates objective function.

Assumption 2a *Candidates choose strategy proposals to maximize their expected utility from corporate office.*

Assumption 2b *Candidates choose strategy proposals to maximize their vote share.*

Additionally, we make the following technical assumptions:

Assumption 3 *The strategy space of each firm j is a nonempty compact subset of $\Re$.*

Assumption 4 *The return of shareholder k is (a) continuous and twice differentiable in the firms' strategies, with continuous second derivatives; and (b) strictly concave in firm j's strategy, conditional on the strategies of the remaining firms.*

Assumption 5 *The random utility ξ_{kj} is distributed uniformly on $\left(-\frac{1}{2}\tau_j\phi_{kj}, \frac{1}{2}\tau_j\phi_{kj}\right)$.*

The key, distinctive, technical assumption is Assumption 5. It implies that managers expect that shareholders with higher financial stakes in the firm will take more interest on their actions and could therefore potentially have a larger preference towards or against the challenger than other shareholders. This is consistent with a significant literature that has examined the incentives of large shareholders to undertake costly monitoring of the firm and intervene to correct the manager's suboptimal decisions (see, e.g., Chidambaran and John, 2003, and references therein). Finally, it implies also that the utility associated to a firm in which a shareholder does not hold financial stakes is null.

The following Proposition characterizes the equilibrium.

[6] This contrasts with Brito *et al.* (2018), who assume that random utility shocks are also identically distributed across firms and shareholders.

Proposition 1 *Under Assumptions* 1, 2*a or* 2*b*, 3, 4 *and* 5, *there exists a pure-strategy Nash equilibrium for the candidates strategy proposals' game where each candidate maximizes*

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\tilde{R}_k \tag{4}$$

Under Assumption 2*a*, γ_{kj} *is measured by the normalized Banzhaf (*1965*) power index of shareholder* k *in firm* j*:* $\gamma_{kj} = \lambda^p_{jk}/\sum_{h\in\Theta}\lambda^p_{jh}$, *where* λ^p_{jk} *denotes the number of subsets of shareholders that can award victory to a candidate in which shareholder* k *is pivotal. Under assumption* 2*b*, γ_{kj} *is measured by the voting rights of shareholder* k *in firm* j*:* $\gamma_{kj} = \upsilon_{kj}$.
Proof. See online appendix B.

Proposition 1 establishes that the manager decides the strategy of the firm to maximize a weighted sum of the firm's shareholders relative returns. The weights γ_{kj} (that are non-negative and sum up to one) capture the importance (or influence) of each shareholder over the decision-making of the firm and are a measure of her control in the firm. This implies the manager maximizes a weighted sum of the profits of (potentially) all firms, $\sum_{g\in\Im} l_{jg}\pi_g$, where the weights $l_{jg} = \sum_{k\in\Theta_j}\gamma_{kj}(\phi_{kg}/\phi_{kj})$ are non-negative.[7]

4 Conclusions

We propose an alternative formulation to model the objective function of the manager of a firm in the presence of horizontal shareholding. In this alternative formulation, the manager decides the strategy of the firm by maximizing a weighted sum of the firm's shareholders relative returns. We do not claim it to be preferred to O&S's formulation. We solely propose it as microfounded alternative which avoids an allegedly unattractive feature of the O&S's formulation: that if non-horizontal shareholders are highly dispersed, managers would mimic the interests of horizontal shareholders even if they own a share of the firm that does not induce full control. Future empirical testing might help establish which formulation more accurately predicts firm behavior. This alternative formulation can be straightforwardly incorporated into the generalized unilateral effects screens proposed in Brito *et al.* (2018).

[7]With cross-ownership among firms, the corresponding weights would be $l_{jg} = \sum_{k\in\Theta_j}\gamma^u_{kj}(\phi^u_{kg}/\phi^u_{kj})$, where ϕ^u_{kj} and γ^u_{kj} denote the *ultimate* financial and control rights, respectively, of external shareholder k in firm j, which can be computed following the algorithm in Brito *et al.* (2018).

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Online Appendix A

In this appendix, we examine properties (v) and (vi) of the manager's objective function for the illustration example in the main text. To do so, consider the horizontal shareholder holds $x < 1$ financial and voting rights in both firms and that each

of the remaining $n > (1-x)/x$ smaller shareholders holds equal financial and voting rights in solely one firm. Let γ_l denote the Banzhaf power index of the horizontal shareholder and $\gamma_s = (1-\gamma_l)/n$ denote the Banzhaf power index of each of the remaining non-horizontal shareholders.

The Banzhaf power index of the horizontal shareholder γ_l is obtained as follows. Consider initially those subsets of shareholders that aggregate more than 50% of the voting rights and that do not include the horizontal shareholder. Each subset must include z smaller non-horizontal shareholders such that $z\frac{1-x}{n} > \frac{1}{2}$ which is equivalent to $z \geq \left\lfloor \frac{n}{2(1-x)} \right\rfloor + 1$, where $\lfloor y \rfloor$ denotes the largest integer lower than y. Any single smaller non-horizontal shareholder is pivotal in one of these subsets if $(z-1)\frac{1-x}{n} < \frac{1}{2}$ which is equivalent to $z \leq \left\lfloor \frac{n}{2(1-x)} + 1 \right\rfloor$.[8] Therefore, any small non-horizontal shareholder is pivotal in all subsets of $z = \left\lfloor \frac{n}{2(1-x)} + 1 \right\rfloor$ small shareholders in which she is present. There are $C_z^n = \frac{n!}{(n-z)!z!}$ different subsets with z smaller non-horizontal shareholders. Therefore, the number of subsets that do not include the horizontal shareholder in which any small non-horizontal shareholder is pivotal is $C^{n-1}_{\left\lfloor \frac{n}{2(1-x)} \right\rfloor + 1 - 1}$.

Consider now those subsets of shareholders that aggregate more than 50% of the voting rights and that include the horizontal shareholder. Each subset must include z smaller non-horizontal shareholders such that $z\frac{1-x}{n} + x > \frac{1}{2}$ which is equivalent to $z \geq \left\lfloor (\frac{1}{2} - x)\frac{n}{1-x} \right\rfloor + 1$. Any single smaller non-horizontal shareholder is pivotal in one of these subsets if $(z-1)\frac{1-x}{n} + x < \frac{1}{2}$ which is equivalent to $z \leq \left\lfloor (\frac{1}{2} - x)\frac{n}{1-x} + 1 \right\rfloor$. Therefore, any small shareholder is pivotal in all subsets that include the horizontal shareholder and $z = \left\lfloor (\frac{1}{2} - x)\frac{n}{1-x} + 1 \right\rfloor$ small shareholders in which she is present. The number of subsets that include the horizontal shareholder in which any small non-horizontal shareholder is pivotal is $C^{n-1}_{\left\lfloor (\frac{1}{2}-x)\frac{n}{1-x} \right\rfloor + 1 - 1}$. In turn, the horizontal shareholder is pivotal in those subsets that include her and z small non-horizontal shareholders if $z\frac{1-x}{n} < \frac{1}{2}$ which is equivalent to $z \leq \left\lfloor \frac{n}{2(1-x)} \right\rfloor$. Therefore, the horizontal shareholder is pivotal in all subsets that include her and z small non-horizontal shareholders, with:

$$\left\lfloor (\frac{1}{2} - x)\frac{n}{1-x} \right\rfloor + 1 \leq z \leq \left\lfloor \frac{n}{2(1-x)} \right\rfloor, \tag{5}$$

which implies the number of sets in which the horizontal shareholder is pivotal is:

$$\sum_{y=\left\lfloor (\frac{1}{2}-x)\frac{n}{1-x} \right\rfloor + 1}^{\left\lfloor \frac{n}{2(1-x)} \right\rfloor} C_y^n. \tag{6}$$

Using the information above, by definition, the Banzhaf power index of the horizontal shareholder is:

$$\gamma_l = \frac{\sum_{y=\left\lfloor (\frac{1}{2}-x)\frac{n}{1-x} \right\rfloor + 1}^{\left\lfloor \frac{n}{2(1-x)} \right\rfloor} C_y^n}{\sum_{y=\left\lfloor (\frac{1}{2}-x)\frac{n}{1-x} \right\rfloor + 1}^{\left\lfloor \frac{n}{2(1-x)} \right\rfloor} C_y^n + n\left(C^{n-1}_{\left\lfloor \frac{n}{2(1-x)} \right\rfloor} + C^{n-1}_{\left\lfloor (\frac{1}{2}-x)\frac{n}{1-x} \right\rfloor} \right)}. \tag{7}$$

Next, we discuss properties (v) and (vi) under the two formulations. To do so, we make use of the weight that each manager assigns to the profit of the rival firm. It is straightforward to show that this weight is given by the expressions presented in Table A1.

[8] Please see Online Appendix B for the formal definition of pivotal.

Table A1

Weight that each Manager Assigns to the Profit of the Rival Firm

	O&S's Formulation	Proposed Alternative Formulation
Proportional Control	$\frac{x^2}{x^2+(1-x)^2/n}$	x
Banzhaf Control	$\frac{\gamma_l x}{\gamma_l x+(1-\gamma_l)(1-x)/n}$	γ_l

Property (*v*).

In this setting, the relative size of the horizontal shareholder depends both on its absolute size and on the number of smaller non-horizontal shareholders. In order to examine the impact of the relative size of the horizontal shareholder on the weight that each manager assigns to the profit of the rival firm, we examine how this weight is impacted by x and n, since that relative size increases in x and n.

We begin by examining O&S's formulation. Under proportional control, the weight that each manager assigns to the profit of the rival firm is given by $w_{pc} = \frac{x^2}{x^2+(1-x)^2/n}$, which increases in x and n:

$$\begin{aligned} \frac{\partial w_{pc}}{\partial x} &= 2nx\frac{(1-x)}{(nx^2-2x+x^2+1)^2} > 0 \\ \frac{\partial w_{pc}}{\partial n} &= x^2\frac{(x-1)^2}{(nx^2-2x+x^2+1)^2} > 0. \end{aligned} \tag{8}$$

This implies that property (v) is present both if the number of shareholders is fixed and if it is allowed to vary.

Under Banzhaf control, the weight that each manager assigns to the profit of the rival firm is given by $w_{bc} = \frac{\gamma_l x}{\gamma_l x+(1-\gamma_l)(1-x)/n}$, which increases in x and n if:

$$\begin{aligned} \frac{dw_{bc}}{dx} &= \frac{\partial w_{bc}}{\partial x} + \frac{\partial w_{bc}}{\partial \gamma_l}\frac{\partial \gamma_l}{\partial x} > 0 \\ \frac{dw_{bc}}{dn} &= \frac{\partial w_{bc}}{\partial n} + \frac{\partial w_{bc}}{\partial \gamma_l}\frac{\partial \gamma_l}{\partial n} > 0. \end{aligned}$$

As:

$$\begin{aligned} \frac{\partial w_{bc}}{\partial n} &= \frac{x\gamma_l(1-\gamma_l)(1-x)}{(-x-\gamma_l+x\gamma_l+nx\gamma_l+1)^2} > 0 \\ \frac{\partial w_{bc}}{\partial x} &= \frac{n\gamma_l(1-\gamma_l)}{(-x-\gamma_l+x\gamma_l+nx\gamma_l+1)^2} > 0 \\ \frac{\partial w_{bc}}{\partial \gamma_l} &= \frac{nx(1-x)}{(-x-\gamma_l+x\gamma_l+nx\gamma_l+1)^2} > 0, \end{aligned} \tag{9}$$

we need to evaluate the sign of $\frac{\partial \gamma_l}{\partial x}$ and $\frac{\partial \gamma_l}{\partial n}$ in order to evaluate the above conditions. We begin by the latter. It is possible to show with examples that $\frac{\partial \gamma_l}{\partial n}$ can take any sign. Consider, for example, the case with $x = 0.05$. When n increases from 499 to 500, γ_l increases 0.016 whereas when n increases from 500 to 501, γ_l decreases 0.016. This implies that property (v) may or may not be present if the number of shareholders is allowed to vary, depending on the number of shareholders and the corresponding subsets in which each shareholder is pivotal. We now address the former. As $\frac{\partial((1/2-x)n/(1-x))}{\partial x} = -\frac{n}{2(1-x)^2} < 0$ and $\frac{\partial(n/2(1-x))}{\partial x} = \frac{n}{2(1-x)^2} > 0$, the number of terms in the summation term that determines the number of subsets in which the horizontal shareholder is pivotal cannot decrease with x. We now consider the effect of x in the number of subsets in which any small shareholder is pivotal, starting with $C^{n-1}_{\lfloor(\frac{1}{2}-x)\frac{n}{1-x}\rfloor}$. By construction, C^{n-1}_p increases in p until $p = (n-1)/2$ and then

decreases. As $\frac{(n-1)/2-(1/2-x)n}{(1-x)} = \frac{x(n+1)-1}{2(1-x)} > 0$ for all x (otherwise, the horizontal shareholder is smaller than the others), we can conclude that increasing x may maintain or decrease $\left\lfloor(\frac{1}{2}-x)\frac{n}{1-x}\right\rfloor$, which never increases $C^{n-1}_{(\frac{1}{2}-x)\frac{n}{1-x}}$. We now turn to $C^{n-1}_{\frac{n}{2(1-x)}}$. As $\frac{(n-1)}{2} - \frac{n}{2(1-x)} = -\frac{nx+1-x}{2(1-x)} < 0$ for all x, increasing x may maintain or increase $\left\lfloor\frac{n}{2(1-x)}\right\rfloor$ which never increases $C^{n-1}_{\left\lfloor\frac{n}{2(1-x)}\right\rfloor}$. This implies that increases in x cannot lead to a decrease in the Banzhaf power index and, hence, cannot decrease w_{bc}. This implies that property (v) is present if the number of shareholders of a firm is fixed.

We now examine the proposed alternative formulation. Under proportional control, the weight that each manager assigns to the profit of the rival firm is given by x, which increases in x and does not vary in n. This implies that property (v) is present if the number of shareholders is fixed while it is not present if the number of shareholders is allowed to vary. Under Banzhaf control, the weight that each manager assigns to the profit of the rival firm is given by γ_l. As discussed in the O&S's formulation section above, this may decrease with n but never decreases with x. This implies that property (v) is present if the number of shareholders is fixed while it may be present or not if the number of shareholders is allowed to vary, depending on the number of shareholders and the corresponding subsets in which each shareholder is pivotal.

Property (vi).

Under proportional control, the difference in weights that each manager assigns to the profit of the rival firm between O&S's formulation and our proposed alternative formulation is given by:

$$\frac{x^2}{x^2+\frac{(1-x)^2}{n}} - x = x\,(1-x)\,\frac{x\,(n+1)-1}{x^2\,(n+1)-2x+1}. \tag{10}$$

As $x^2\,(n+1)-2x+1 > x^2-2x+1 = (1-x)^2 > 0$ this is always positive, meaning the weights that each manager assigns to the profit of the rival firm are lower under the proposed alternative formulation than under O&S's formulation. Moreover, the weight that each manager assigns to the profit of the rival firm under O&S's formulation tends to one, for any given value of the horizontal shareholder's control rights x, as $n \to \infty$. In contrast, the weight that each manager assigns to the profit of the rival firm under the proposed alternative formulation, solely tends to one when x tends to one.

Under Banzhaf control, the difference in weights that each manager assigns to the profit of the rival firm between O&S's formulation and our proposed alternative formulation is given by:

$$\frac{\gamma_l x}{\gamma_l x+\frac{(1-\gamma_l)(1-x)}{n}} - \gamma_l = \frac{\gamma_l\,(1-\gamma_l)\,(x\,(n+1)-1)}{\gamma_l\,(x\,(n+1)-1)+1-x} > 0, \tag{11}$$

which is always positive, meaning the weights that each manager assigns to the profit of the rival firm are lower under the proposed alternative formulation than under O&S's formulation. Moreover, the weight that each manager assigns to the profit of the rival firm under O&S's formulation tends to one, for any given value of the horizontal shareholder's control rights γ_l, as $n \to \infty$. In contrast, the weight that each manager assigns to the profit of the rival firm under the proposed alternative formulation, solely tends to one when γ_l tends to one.

Online Appendix B

In this appendix, we present the proof of Proposition 1. We divide it in two. We first present the proof under Assumptions 1, 2a, 3, 4 and 5. We then present the proof under Assumptions 1, 2b, 3, 4 and 5. To do so, let x_{aj} and x_{bj} denote the strategy

proposals of the incumbent and the challenger for firm j, respectively and let $\mathbf{x} = (x_1, \ldots, x_j, \ldots, x_N)^\top$ denote the $N \times 1$ vector of strategy proposals for all the firms in the industry. Further, let $m_j \in \{a_j, b_j\}$ denote the identity of the candidate that receives the majority of firm j's voting rights (being elected manager of the firm) and let $\mathbf{m} = (m_1, \ldots, m_j, \ldots, m_N)^\top$ denote the $N \times 1$ vector of elected managers for all the firms in the industry. Finally, let $u_k(\mathbf{x}, \mathbf{m})$ and $R_k(\mathbf{x})$ express the mathematical dependence of the utility and return of shareholder k, respectively, on the winning strategies of all firms.

Proof of Proposition 1 under Assumptions 1, 2a, 3, 4 and 5.

Assumption 2a implies that candidates choose strategy proposals so to maximize the product of the probability that they are elected in the second stage and the utility obtained from the rent associated with corporate office they expect to accrue conditional upon being elected, which we denote Ξ. Since the maximization problem of the two candidates to firm j is symmetric, we describe - for simplicity of exposition - solely the incumbent's problem, who chooses x_{aj} so to solve:

$$\max_{x_{aj}} \Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)\, \Xi, \tag{12}$$

where $\mathbf{x}_a = (x_1, \ldots, x_{aj}, \ldots, x_N)^\top$, $\mathbf{x}_b = (x_1, \ldots, x_{bj}, \ldots, x_N)^\top$ and $\Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$ denotes the probability with which the incumbent is, from the candidates perspective, elected manager of the firm in the second stage.

In order to solve the above maximization problem, we must beforehand derive $\Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$. To do so, let ℓ_j denote the number of shareholders with voting rights in firm j, $\wp_j$ denote all the $2^{\ell_j - 1}$ possible subsets of those shareholders that can award the majority of votes to a candidate and $\Theta_j^\imath \in \wp_j$ denote a particular subset of those shareholders. Given that the incumbent's election is ensured with the votes of all shareholders in each subset in $\wp_j$, we have that the probability $\Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$ with which she is elected manager of firm j just sums the probabilities $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right)$ with which she is elected in each subset $\Theta_j^\imath$, as follows:

$$\begin{aligned} \Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b) &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right) \\ &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k \in \Theta_j^\imath} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) \prod\nolimits_{k \notin \Theta_j^\imath} \Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) \\ &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k \in \Theta_j^\imath} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) \prod\nolimits_{k \notin \Theta_j^\imath} \left(1 - \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\right), \end{aligned} \tag{13}$$

where $\mathbf{m}_a = (m_1, \ldots, a_j, \ldots, m_N)^\top$ and $\mathbf{m}_b = (m_1, \ldots, b_j, \ldots, m_N)^\top$ while $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ and $\Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) = 1 - \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ denote the probability that shareholder k votes for the incumbent and the challenger, respectively. It remains to derive $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$, which is given by:

$$\begin{aligned} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) &= \Pr(u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)) \\ &= \Pr\left(R_k(\mathbf{x}_a) > R_k(\mathbf{x}_b) + \xi_{jg}\right) \\ &= \Pr\left(\xi_{jg} < R_k(\mathbf{x}_a) - R_k(\mathbf{x}_b)\right) \\ &= G_{kj}(R_k(\mathbf{x}_a) - R_k(\mathbf{x}_b)) \\ &= \frac{1}{2} + \frac{1}{\tau_j \phi_{kj}} (R_k(\mathbf{x}_a) - R_k(\mathbf{x}_b)), \end{aligned} \tag{14}$$

where the second equality makes use of the fact that the term $\sum_{g \in \Im \setminus j} d_g \xi_{kg}$ enters the utility obtained from both strategy proposals and the last equality makes use of Assumption 5. This implies that we can rewrite the incumbent's problem as follows:

$$\max_{x_{aj}} \varpi_{aj} = \left(\sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k \in \Theta_j^\imath} \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{k \notin \Theta_j^\imath} \left(1 - \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)\right)\right) \Xi. \tag{15}$$

Assumptions 1, 3, 4 and 5 imply that this problem is strictly concave conditional on rival firm strategies and therefore has a unique maximum. In order to see why note that, under Assumption 1, shareholders are conditionally sincere, which implies that the incumbent candidate to firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial \varpi_{aj}}{\partial x_{aj}} = \sum\nolimits_{k \in \Theta} \frac{\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)} \frac{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}}, \tag{16}$$

where, using probability (13), we have that:

$$\begin{aligned} \frac{\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)} &= \sum\nolimits_{\Theta_j^\imath \in \wp_j, k \in \Theta_j^\imath} \prod\nolimits_{h \in \Theta_j^\imath, h \neq k} \Pr_{ha_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{h \notin \Theta_j^\imath} \left(1 - \Pr_{ha_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)\right) \\ &\quad - \sum\nolimits_{\Theta_j^\imath \in \wp_j, k \notin \Theta_j^\imath} \prod\nolimits_{h \in \Theta_j^\imath} \Pr_{ha_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{h \notin \Theta_j^\imath, h \neq k} \left(1 - \Pr_{ha_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)\right), \end{aligned} \tag{17}$$

which (i) is, by definition, non-negative since increasing $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ for any k can not have a negative impact on $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)$; and (ii) does not depend on $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ since $\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)$ is linear in $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$ for any k taking the corresponding probabilities of the remaining shareholders as given. The second order condition of the incumbent's problem, in turn, is given by:

$$\begin{aligned} \frac{\partial^2 \varpi_{aj}}{\partial x_{aj}^2} &= \sum\nolimits_{k \in \Theta_j} \frac{\partial^2 \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \partial x_{aj}} \frac{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}} \\ &\quad + \sum\nolimits_{k \in \Theta_j} \frac{\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)} \frac{\partial^2 \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}^2} \\ &= \sum\nolimits_{k \in \Theta_j} \frac{\partial^2 \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)^2} \left(\frac{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}}\right)^2 \\ &\quad + \sum\nolimits_{k \in \Theta_j} \frac{\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)} \frac{\partial^2 \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}^2} \\ &= \sum\nolimits_{k \in \Theta_j} \frac{\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)} \frac{\partial^2 \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)}{\partial x_{aj}^2}, \end{aligned} \tag{18}$$

where the last equality makes use of the fact that $\partial^2 \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) / \partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)^2 = 0$ for all k, since it does not depend on $\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)$. Using probability (13), we have that the objective function of the manager is strictly concave in x_{aj}, conditional on the strategies of the remaining firms, since (i) $\partial^2 \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) / \partial x_{aj}^2 = \left(1/\tau_j \phi_{kj}\right) \partial^2 R_k\left(\mathbf{x}_a\right) / \partial x_{aj}^2$ is negative under Assumption 4; and (ii) $\partial \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) / \partial \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) > 0$ for at least an shareholder k. Finally, given that the strategy proposal x_{aj} is, under Assumption 3, defined in a convex set, we have that the incumbent's maximization problem has an unique maximum.

Given the symmetry of the maximization problem of the challenger candidate to firm j, we have that the two candidates will choose the same best-response function, i.e., the same strategy proposal for the firm, conditional on the strategies of the candidates to the remaining firms. We now show that this best-response function is the same as the best-response function that arises while maximizing a weighted average of the relative returns of the firm's shareholders conditional on the strategies of the candidates to the remaining firms, with normalized Banzhaf power indices as weights. To do so, note that since the two candidates will choose the same best-response function, in equilibrium, we have $R_k(\mathbf{x}_a) = R_k(\mathbf{x}_b) = R_k(\mathbf{x})$ for all k. This

implies that the first-order condition reduces to:

$$\frac{1}{2^{\ell_j-1}}\sum\nolimits_{\Theta^{\imath}_j\in\wp_j}\sum\nolimits_{k\in\Theta^{\imath}_j}\frac{1}{\tau_j\phi_{kj}}\frac{\partial R_k(\mathbf{x})}{\partial x_j}-\frac{1}{2^{\ell_j-1}}\sum\nolimits_{\Theta^{\imath}_j\in\wp_j}\sum\nolimits_{k\notin\Theta^{\imath}_j}\frac{1}{\tau_j\phi_{kj}}\frac{\partial R_k(\mathbf{x})}{\partial x_j}\leqslant 0, \tag{19}$$

which makes use of the fact that $\partial \Pr_{ka_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b)/\partial x_{aj} = (1/\tau_j\phi_{kj})\,\partial R_k(\mathbf{x}_a)/\partial x_{aj}$ and $\Pr_{ka_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b) = \Pr_{kb_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b) = 1/2$ when $\mathbf{x}_a=\mathbf{x}_b$, both for all k. This first-order condition can, in turn, be rewritten as:

$$\frac{1}{2^{\ell_j-1}}\sum\nolimits_{k\in\Theta_j}\left(\lambda_{jk}\frac{1}{\tau_j\phi_{kj}}\frac{\partial R_k(\mathbf{x})}{\partial x_j}-\left(2^{\ell_j-1}-\lambda_{jk}\right)\frac{1}{\tau_j\phi_{kj}}\frac{\partial R_k(\mathbf{x})}{\partial x_j}\right)\leqslant 0, \tag{20}$$

where λ_{jk} denotes the number of subsets in $\wp_j$ in which shareholder k enters and $\left(2^{\ell_j-1}-\lambda_{jk}\right)$ denotes the number of subsets in $\wp_j$ in which shareholder k does not enter. Finally, consider that λ_{jk} can be divided in two terms: the number of subsets in $\wp_j$ in which shareholder k enters and is pivotal, λ^{p}_{jk}, and the number of subsets in $\wp_j$ in which shareholder k enters and is not pivotal, $\lambda^{\tilde{p}}_{jk}$.[9] The latter is, by construction, equal to the number of subsets in $\wp_j$ in which shareholder k does not enter. This implies that $\lambda^{\tilde{p}}_{jk}=\left(2^{\ell_j-1}-\lambda_{jk}\right)$ and that the first-order condition can be rewritten as:

$$\sum\nolimits_{k\in\Theta_j}\left(\frac{\lambda^{p}_{jk}}{2^{\ell_j-1}}\right)\frac{1}{\tau_j\phi_{kj}}\frac{\partial R_k(\mathbf{x})}{\partial x_j}\leqslant 0, \tag{21}$$

where $\lambda^{p}_{jk}/2^{\ell_j-1}$ denotes the Banzhaf power index associated to shareholder k in firm j. This establishes that, in equilibrium, the candidates to each firm converge to the same strategy, which also maximizes the following weighted average of the relative returns of the shareholders with voting rights in the firm, conditional on the strategies of the candidates to the remaining firms:

$$\max_{x_j}\sum\nolimits_{k\in\Theta_j}\gamma_{kj}\tilde{R}_k(\mathbf{x}), \tag{22}$$

where $\gamma_{kj} = \left(\lambda^{p}_{jk}/2^{\ell_j-1}\right)/\sum_{k\in\Theta}\left(\lambda^{p}_{jk}/2^{\ell_j-1}\right) = \lambda^{p}_{jk}/\sum_{k\in\Theta}\lambda^{p}_{jk}$ denotes the weight assigned by firm j's manager to the return of shareholder k, measured by the normalized Banzhaf power index of shareholder k in firm j.

Finally, given that the strategy proposal of each candidate to the different firms is, under Assumption 3, defined in a convex set and $R_k(\mathbf{x})$ is, under Assumption 4, continuous, the best-response functions of the candidates to the different firms are guaranteed to be upper-hemicontinuous, which implies that we can apply Kakutani's fixed point theorem to ensure that the Nash equilibrium exists.

Proof of Proposition 1 under Assumptions 1, 2b, 3, 4 and 5.

Assumption 2*b* implies that candidates choose strategy proposals so to maximize the sum, across all shareholders, of the product of the probability that each shareholder votes for the candidate by the corresponding voting rights. Again, since the maximization problem of the two candidates to firm j is symmetric, we describe - for simplicity of exposition - solely the incumbent's problem, who chooses x_{aj} so to solve:

$$\max_{x_{aj}}\sum\nolimits_{k\in\Theta_j}\Pr\nolimits_{ka_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b)\,\upsilon_{kj}=\sum\nolimits_{k\in\Theta_j}\left(\frac{1}{2}+\left(\frac{1}{\tau_j\phi_{kj}}\right)(R_k(\mathbf{x}_a)-R_k(\mathbf{x}_b))\right)\upsilon_{kj}, \tag{23}$$

where, under Assumption 5, $\Pr_{ka_j}(\mathbf{x}_a,\mathbf{m}_a,\mathbf{x}_b,\mathbf{m}_b)=(1/2)+\left(1/\tau_j\phi_{kj}\right)(R_k(\mathbf{x}_a)-R_k(\mathbf{x}_b))$ denotes, as discussed above, the

[9] Shareholder k is pivotal if for some subset $\Theta^{\imath}_j$ which does not include shareholder k, we have $\sum_{h\in\Theta^{\imath}_j}\upsilon_{hj}\leq 0.5$, but if we include shareholder k, $\sum_{h\in\Theta^{\imath}_j}\upsilon_{hj}>0.5$.

probability that shareholder k votes for the incumbent.

Assumptions 1, 3, 4 and 5 imply that this problem is strictly concave conditional on rival firm strategies and therefore has a unique maximum. In order to see why note that, under Assumption 1, shareholders are conditionally sincere, which implies that the incumbent candidate to firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial \varpi_{aj}}{\partial x_{aj}} = \sum\nolimits_{k \in \Theta_j} \frac{1}{\tau_j \phi_{kj}} \frac{\partial R_k\left(\mathbf{x}_a\right)}{\partial x_{aj}} \upsilon_{kj}, \tag{24}$$

whereas the second order condition is given by:

$$\frac{\partial^2 \varpi_{aj}}{\partial x_{aj}^2} = \sum\nolimits_{k \in \Theta_j} \frac{1}{\tau_j \phi_{kj}} \frac{\partial^2 R_k\left(\mathbf{x}_a\right)}{\partial x_{aj}^2} \upsilon_{kj}, \tag{25}$$

which implies, given Assumption 4, that the objective function of the manager is strictly concave in x_{aj}, conditional on the strategies of the remaining firms. Finally, given that the strategy proposal x_{aj} is, under Assumption 3, defined in a convex set, we have that the incumbent's maximization problem has an unique maximum.

Given the symmetry of the maximization problem of the challenger candidate to firm j, we have that the two candidates will choose the same best-response function, i.e., the same strategy proposal for the firm, conditional on the strategies of the candidates to the remaining firms. This establishes that, in equilibrium, the candidates to each firm converge to the same strategy, which also maximizes the following weighted average of the relative returns of the shareholders with voting rights in the firm, conditional on the strategies of the candidates to the remaining firms:

$$\max_{x_j} \sum\nolimits_{k \in \Theta_j} \gamma_{kj} \tilde{R}_k\left(\mathbf{x}\right), \tag{26}$$

where $\gamma_{kj} = \upsilon_{kj}$ denotes the weight assigned by firm j's manager to the return of shareholder k, measured by the voting rights of shareholder k in firm j.

Finally, given that the strategy proposal of each candidate to the different firms is, under Assumption 3, defined in a convex set and $R_k(\mathbf{x})$ is, under Assumption 4, continuous, the best-response functions of the candidates to the different firms are guaranteed to be upper-hemicontinuous, which implies that we can apply Kakutani's fixed point theorem to ensure that the Nash equilibrium exists.